

AREA BETWEEN 2 FUNCTIONS

When finding the area enclosed between two functions:

Step 1: Sketch the curves

Step 2: Find the points of intersections

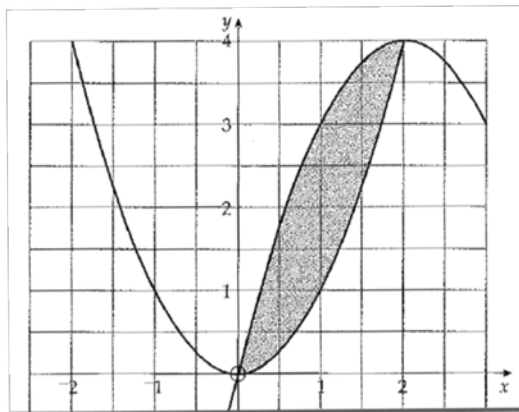
Step 3: Form a function which is the difference between the two functions [i.e top function $f(x)$ minus bottom function $g(x)$]

Step 4: Integrate $f(x) - g(x)$, the limits of integration are the points of intersection

Rule: Area enclosed between the top function $f(x)$ and $g(x)$, where the points of intersection are a and b , is $A = \int_a^b f(x) - g(x) dx$.

* This rule works even if part of the area enclosed between the 2 functions is below the x -axis.

Example: Find the area enclosed between the two functions $y = x^2$ and $y = 4x - x^2$.



Top function is $f(x) = 4x - x^2$, bottom function is $g(x) = x^2$.

Difference between the two functions:

$$\begin{aligned} f(x) - g(x) &= (4x - x^2) - (x^2) \\ &= 4x - x^2 - x^2 \\ &= 4x - 2x^2 \end{aligned}$$

The points of intersection are $x = 0$ and $x = 2$.

Area enclosed between the two functions:

$$\begin{aligned} A &= \int_0^2 4x - 2x^2 dx \\ &= \left[\frac{4}{2}x^2 - \frac{2}{3}x^3 \right]_0^2 = \left[2x^2 - \frac{2}{3}x^3 \right]_0^2 = 2\frac{2}{3} \text{ units}^2 \end{aligned}$$

Delta Ex 20.3 pg 191 – 192

Extenson EX 20 pg 193 - 194